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Lightlike branes as natural candidates for wormhole throats

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We first briefly present a consistent world-volume Lagrangian description of lightlike p-branes (*LL-branes*) in two equivalent forms – a Polyakov-type and a dual to it Nambu-Goto-type formulations. The most important characteristic features of *LL-brane* dynamics are: (i) the brane tension appears as a non-trivial additional dynamical degree of freedom; (ii) consistency of *LL-brane* dynamics in a spherically or axially symmetric gravitational background of codimension one requires the presence of an event horizon which is automatically occupied by the *LL-brane* ("horizon straddling"). Next we consider a bulk Einstein-Maxwell system interacting self-consistently with a codimension one *LL-brane*. We find spherically symmetric traversable wormhole solutions of Misner-Wheeler type produced by the *LL-brane* sitting at the wormhole throat with wormhole parameters being functions of the dynamical *LL-brane* tension.

1 Introduction

The notion of wormhole space-time in the context of Schwarzschild geometry was first introduced by Einstein and Rosen [1] and understood in full details after the work of Kruskal and Szekeres [2]. Einstein and Rosen [1] also discussed the possibility of a singularity free wormhole solution obtained at the price of considering as source a Maxwell field with an energy momentum tensor being the opposite to the standard one. Misner and Wheeler [3] also realized that wormholes connecting two asymptotically flat space times provide the possibility of "charge without charge", i.e., electromagnetically non-trivial solutions where the lines of force of the electric field flow from one universe to the other without a source and giving the impression of being positively charged in one universe and negatively charged in the other universe.

In order to construct wormholes of the traversable type, however, matter which violate the null energy condition must be introduced [4]. An interesting consequence of this construction is that the resulting space time is non-singular. Here we will discuss within this context the role of lightlike p-branes (*LL-branes* for short), whose explicit world-volume Lagrangian formulation was given in our previous papers [5]. As we will see in what follows, *LL-branes* are particularly well suited to serve as gravitational sources of Misner-Wheeler type wormholes located at their throats, while at the same time they obey a well-defined dynamics derivable from first principles.

Let us note that *LL-branes* by themselves attract special interest in general relativity. This is due primarily because of their role in the effective description of many cosmological and astrophysical effects such

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as impulsive lightlike signals arising in cataclysmic astrophysical events [6], the "membrane paradigm" theory of black hole physics [7], the thin-wall approach to domain walls coupled to gravity [8, 9]. More recently *LL-branes* acquired significance also in the context of modern non-perturbative string theory [10].

There are two basic properties of *LL-branes* which drastically distinguish them from ordinary Nambu-Goto branes: (i) they describe intrinsically lightlike modes, whereas Nambu-Goto branes describe massive ones; (ii) the tension of the *LL-brane* arises as an additional dynamical degree of freedom, whereas Nambu-Goto brane tension is a given ad hoc constant.

2 World-volume Lagrangian formulation of lightlike branes

In [5, 11] we have proposed a systematic Lagrangian formulation of a generalized Polyakov-type for *LL-branes* in terms of the world-volume action:

$$S_{\rm LL} = \int d^{p+1}\sigma\, \Phi(\varphi) \left[-\frac{1}{2}\gamma^{ab} g_{ab} + L\big(F^2\big) \right] . \tag{1}$$

Here the following notations are used: $a, b = 0, 1, \ldots, p$; $(\sigma^a) \equiv (\tau, \sigma^i)$ with $i = 1, \ldots, p$; γ_{ab} denotes the intrinsic Remain metric on the world-volume;

$$g_{ab} \equiv \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) \tag{2}$$

is the induced metric (the latter becomes *singular* on-shell – lightlikeness, cf. second Eq. (6) below);

$$\Phi(\varphi) \equiv \frac{1}{(p+1)!} \varepsilon_{I_1 \ldots I_{p+1}} \varepsilon^{a_1 \ldots a_{p+1}} \partial_{a_1} \varphi^{I_1} \ldots \partial_{a_{p+1}} \varphi^{I_{p+1}} \tag{3}$$

is an alternative non-Riemannian reparametrization-covariant integration measure density replacing the standard $\sqrt{-\gamma} \equiv \sqrt{-\det \|\gamma_{ab}\|}$ and built from auxiliary world-volume scalars $\{\varphi^I\}_{I=1}^{p+1}$;

$$F_{a_1 \ldots a_p} = p \partial_{[a_1} A_{a_2 \ldots a_p]} \quad , \quad F^{*a} = \frac{1}{p!} \frac{\varepsilon^{a a_1 \ldots a_p}}{\sqrt{-\gamma}} F_{a_1 \ldots a_p} \tag{4}$$

are the field-strength and its dual one of an auxiliary world-volume $(p-1)$-rank antisymmetric tensor gauge field $A_{a_1 \ldots a_{p-1}}$ with Lagrangian $L(F^2)$ ($F^2 \equiv F_{a_1 \ldots a_p} F_{b_1 \ldots b_p} \gamma^{a_1 b_1} \ldots \gamma^{a_p b_p}$).

Equivalently one can rewrite (1) as:

$$S = \int d^{p+1}\sigma\, \chi \sqrt{-\gamma} \left[-\frac{1}{2}\gamma^{ab} g_{ab} + L\big(F^2\big) \right] \quad , \quad \chi \equiv \frac{\Phi(\varphi)}{\sqrt{-\gamma}} \tag{5}$$

The composite field χ plays the role of a *dynamical (variable) brane tension*.

For the special choice $L\big(F^2\big) = \big(F^2\big)^{1/p}$ the above action becomes invariant under Weyl (conformal) symmetry: $\gamma_{ab} \longrightarrow \gamma'_{ab} = \rho\, \gamma_{ab}$, $\varphi^i \longrightarrow \varphi'^{\,i} = \varphi'^{\,i}(\varphi)$ with Jacobian $\det \Big\| \frac{\partial \varphi'^{\,i}}{\partial \varphi^j} \Big\| = \rho$.

Consider now the equations of motion corresponding to (1) w.r.t. φ^I and γ^{ab}:

$$\frac{1}{2}\gamma^{cd} g_{cd} - L(F^2) = M \quad , \quad \frac{1}{2} g_{ab} - F^2 L'(F^2) \left[\gamma_{ab} - \frac{F^*_a F^*_b}{F^{*\,2}} \right] = 0 . \tag{6}$$

Here M is an integration constant and $F^{*\,a}$ is the dual field strength (4). Both Eqs. (6) imply the constraint $L\big(F^2\big) - pF^2 L'\big(F^2\big) + M = 0$, i.e.

$$F^2 = F^2(M) = \text{const on} - \text{shell} . \tag{7}$$

The second Eq. (6) exhibits *on-shell singularity* of the induced metric (2):

$$g_{ab}F^{*\,b} = 0 \; , \tag{8}$$

i.e., the tangent vector to the world-volume $F^{*\,a}\partial_a X^\mu$ is *lightlike* w.r.t. metric of the embedding space-time.

Further, the equations of motion w.r.t. world-volume gauge field $A_{a_1 \ldots a_{p-1}}$ (with χ as defined in (5) and accounting for the constraint (7)) $\partial_{[a}\left(F^*_{b]}\,\chi\right) = 0$ allow us to introduce the dual "gauge" potential u:

$$F^*_a = \text{const}\,\frac{1}{\chi}\partial_a u \; . \tag{9}$$

We can rewrite second Eq. (6) (the lightlike constraint) in terms of the dual potential u as:

$$\gamma_{ab} = \frac{1}{2a_0}\,g_{ab} - \frac{2}{\chi^2}\,\partial_a u \partial_b u \quad , \quad a_0 \equiv F^2 L'\left(F^2\right)\big|_{F^2=F^2(M)} = \text{const} \tag{10}$$

($L'(F^2)$ denotes derivative of $L(F^2)$ w.r.t. the argument F^2). From (9) and (7) we have the relation:

$$\chi^2 = -2\gamma^{ab}\partial_a u \partial_b u \; , \tag{11}$$

and the Bianchi identity $\nabla_a F^{*\,a} = 0$ becomes:

$$\partial_a\Big(\frac{1}{\chi}\sqrt{-\gamma}\gamma^{ab}\partial_b u\Big) = 0 \; . \tag{12}$$

Finally, the X^μ equations of motion produced by the (1) read:

$$\partial_a\left(\chi\sqrt{-\gamma}\gamma^{ab}\partial_b X^\mu\right) + \chi\sqrt{-\gamma}\gamma^{ab}\partial_a X^\nu \partial_b X^\lambda \Gamma^\mu_{\nu\lambda}(X) = 0 \tag{13}$$

where $\Gamma^\mu_{\nu\lambda} = \frac{1}{2}G^{\mu\kappa}\left(\partial_\nu G_{\kappa\lambda} + \partial_\lambda G_{\kappa\nu} - \partial_\kappa G_{\nu\lambda}\right)$ is the Christoffel connection for the external metric.

It is now straightforward to prove that the system of equations (11)–(13) for (X^μ, u, χ), which are equivalent to the equations of motion (6)–(9),(13) resulting from the original Polyakov-type *LL-brane* action (1), can be equivalently derived from the following *dual* Nambu-Goto-type world-volume action:

$$S_{\text{NG}} = -\int d^{p+1}\sigma\, T\sqrt{-\det\|g_{ab} - \frac{1}{T^2}\partial_a u \partial_b u\|} \; . \tag{14}$$

Here g_{ab} is the induced metric (2); T is *dynamical* tension simply related to the dynamical tension χ from the Polyakov-type formulation (5) as $T^2 = \frac{\chi^2}{4a_0}$ with a_0 – same constant as in (10).

Henceforth we will consider the initial Polyakov-type form (1) of the *LL-brane* world-volume action. Invariance under world-volume reparametrizations allows to introduce the standard synchronous gauge-fixing conditions:

$$\gamma^{0i} = 0 \;\; (i = 1, \ldots, p) \; , \; \gamma^{00} = -1 \tag{15}$$

Also, in what follows we will use a natural ansatz for the "electric" part of the auxiliary world-volume gauge field-strength:

$$F^{*i} = 0 \;\; (i = 1, \ldots, p) \quad , \quad \text{i.e.} \;\; F_{0 i_1 \ldots i_{p-1}} = 0 \; . \tag{16}$$

3 Lightlike branes in gravitational backgrounds: codimension one

Let us consider codimension one *LL-brane* moving in a general spherically symmetric background:

$$ds^2 = -A(t,r)(dt)^2 + B(t,r)(dr)^2 + C(t,r)h_{ij}(\vec{\theta})d\theta^i d\theta^j \ , \tag{17}$$

i.e., $D = (p+1)+1$, with the simplest non-trivial ansatz for the *LL-brane* embedding coordinates $X^\mu(\sigma)$:

$$t = \tau \equiv \sigma^0 \quad , \quad r = r(\tau) \quad , \quad \theta^i = \sigma^i \ (i = 1,\ldots,p) \ . \tag{18}$$

The *LL-brane* equations of motion (10)–(13), taking into account (15)–(16), acquire the form:

$$-A + B\,\dot{r}^2 = 0 \ , \ \text{i.e.} \quad \dot{r} = \pm\sqrt{\frac{A}{B}} \quad , \quad \partial_t C + \dot{r}\,\partial_r C = 0 \tag{19}$$

$$\partial_\tau \chi + \chi \left[\partial_t \ln\sqrt{AB} \pm \frac{1}{\sqrt{AB}}\Big(\partial_r A + p\,a_0 \partial_r \ln C\Big)\right]_{r=r(\tau)} = 0 \ , \tag{20}$$

where a_0 is the same constant appearing in (10). In particular, we are interested in static spherically symmetric metrics in standard coordinates:

$$ds^2 = -A(r)(dt)^2 + A^{-1}(r)(dr)^2 + r^2 h_{ij}(\vec{\theta})d\theta^i d\theta^j \tag{21}$$

for which Eqs. (19) yield:

$$\dot{r} = 0 \ , \ \text{i.e.} \ r(\tau) = r_0 = \text{const} \quad , \quad A(r_0) = 0 \ . \tag{22}$$

Eq. (22) tells us that consistency of *LL-brane* dynamics in a spherically symmetric gravitational background of codimension one requires the latter to possess a horizon (at some $r = r_0$), which is automatically occupied by the *LL-brane* ("horizon straddling"). Further, Eq. (20) implies for the dynamical tension:

$$\chi(\tau) = \chi_0 \exp\left\{\mp\tau\left(\partial_r A\,\big|_{r=r_0} + \frac{2p\,a_0}{r_0}\right)\right\} \quad , \quad \chi_0 = \text{const} \ . \tag{23}$$

Thus, we find a time-asymmetric solution for the dynamical brane tension which (upon appropriate choice of the signs ($\mp$) depending on the sign of the constant factor in the exponent on the r.h.s. of (23)) *exponentially "inflates" or "deflates"* for large times (for details we refer to the last two references in [11]). This phenomenon is an analog of the "mass inflation" effect around black hole horizons [12].

Let us note, that similar results ("horizon straddling" and "inflation" of the brane tension) have been obtained also for *LL-brane* moving in axially symmetric Kerr-Newman background [13].

4 Self-consistent Misner-Wheeler traversable wormhole solutions

Let us now consider a self-consistent bulk Einstein-Maxwell system free of electrically charged matter, coupled to a codimension one *LL-brane*:

$$S = \int d^D x\,\sqrt{-G}\,\left[\frac{R(G)}{16\pi} - \frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}\right] + S_{\rm LL} \ . \tag{24}$$

Here $\mathcal{F}_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu$ and $S_{\rm LL}$ is the same *LL-brane* world-volume action as in (5). In other words, the *LL-brane* will serve as a gravitational source through its energy-momentum tensor (see Eq. (26) below). The pertinent Einstein-Maxwell equations of motion read:

$$R_{\mu\nu} - \frac{1}{2}G_{\mu\nu}R = 8\pi\left(T^{(EM)}_{\mu\nu} + T^{(\rm brane)}_{\mu\nu}\right) \quad , \quad \partial_\nu\left(\sqrt{-G}G^{\mu\kappa}G^{\nu\lambda}\mathcal{F}_{\kappa\lambda}\right) = 0 \ , \tag{25}$$

where $T^{(EM)}_{\mu\nu} = \mathcal{F}_{\mu\kappa}\mathcal{F}_{\nu\lambda}G^{\kappa\lambda} - G_{\mu\nu}\frac{1}{4}\mathcal{F}_{\rho\kappa}\mathcal{F}_{\sigma\lambda}G^{\rho\sigma}G^{\kappa\lambda}$, and the *LL-brane* energy-momentum tensor is straightforwardly derived from (5):

$$T^{(\rm brane)}_{\mu\nu} = -G_{\mu\kappa}G_{\nu\lambda}\int d^{p+1}\sigma\, \frac{\delta^{(D)}\big(x - X(\sigma)\big)}{\sqrt{-G}}\,\chi\,\sqrt{-\gamma}\gamma^{ab}\partial_a X^\kappa \partial_b X^\lambda \;, \tag{26}$$

Using (26) we will now construct a traversable *wormhole* solution to the Einstein equations (25) of Misner-Wheeler type [3] following the standard procedure [4]. In other words, the *LL-brane* will serve as a gravitational source of the wormhole by locating itself on its throat.

To this end let us take a spherically symmetric solution of (25) of the form (21) in the absence of the *LL-brane* (i.e., without $T^{(\rm brane)}_{\mu\nu}$ on the r.h.s.), which possesses an event horizon at some $r = r_0$ (i.e., $A(r_0) = 0$). Consider now the following modification of the metric (21):

$$ds^2 = -\widetilde{A}(\eta)(dt)^2 + \widetilde{A}^{-1}(\eta)(d\eta)^2 + (r_0+|\eta|)^2 h_{ij}(\vec{\theta})d\theta^i d\theta^j \quad , \quad \widetilde{A}(\eta) \equiv A(r_0+|\eta|) \;, \tag{27}$$

where $-\infty < \eta < \infty$. From now on the bulk space-time indices μ,ν will refer to (t,η,θ^i). The new metric (27) represents two identical copies of the exterior region ($r > r_0$) of the spherically symmetric space-time with metric (21), which are sewed together along the horizon $r = r_0$. We will show that the new metric (27) is a solution of the full Einstein equations (25), *including* $T^{(\rm brane)}_{\mu\nu}$ on the r.h.s.. Here the newly introduced coordinate η will play the role of a radial-like coordinate normal w.r.t. the *LL-brane* located on the horizon, which interpolates between two copies of the exterior region of (21) (the two copies transform into each other under the "parity" transformation $\eta \to -\eta$).

Inserting in (26) the expressions for $X^\mu(\sigma)$ from (18) and (22) and taking into account (10), (15)–(16) we get:

$$T^{\mu\nu}_{(\rm brane)} = S^{\mu\nu}\,\delta(\eta) \tag{28}$$

with surface energy-momentum tensor:

$$S^{\mu\nu} \equiv -\frac{\chi}{(2a_0)^{p/2-1}}\,\Big[-\partial_\tau X^\mu \partial_\tau X^\nu + \gamma^{ij}\partial_i X^\mu \partial_j X^\nu\Big]_{t=\tau,\,\eta=0,\,\theta^i=\sigma^i} \quad , \quad \partial_i \equiv \frac{\partial}{\partial\sigma^i} \;, \tag{29}$$

where again a_0 is the integration constant parameter appearing in the *LL-brane* dynamics (cf. Eq. (10)). Let us also note that unlike the case of test *LL-brane* moving in a spherically symmetric background (Eqs. (20) and (23)), the dynamical brane tension χ in Eq. (29) is *constant*. This is due to the fact that in the present context we have a discontinuity in the Christoffel connection coefficients across the *LL-brane* sitting on the horizon ($\eta = 0$). The problem in treating the geodesic *LL-brane* equations of motion (13), in particular – Eq. (20), can be resolved following the approach in [8] (see also the regularization approach in ref. [14], Appendix A) by taking the mean value of the pertinent non-zero Christoffel coefficients across the discontinuity at $\eta = 0$. From the explicit form of Eq. (20) it is straightforward to conclude that the above mentioned mean values around $\eta = 0$ vanish since now ∂_r is replaced by $\partial/\partial\eta$, whereas the metric coefficients depend explicitly on $|\eta|$. Therefore, in the present case Eq. (20) is reduced to $\partial_\tau\chi = 0$.

Let us now separate in (25) explicitly the terms contributing to δ-function singularities (these are the terms containing second derivatives w.r.t. η, bearing in mind that the metric coefficients in (27) depend on $|\eta|$):

$$\begin{aligned} R_{\mu\nu} &\equiv \partial_\eta \Gamma^\eta_{\mu\nu} + \partial_\mu\partial_\nu \ln\sqrt{-G} + \text{non}-\text{singular terms} \\ &= 8\pi\left(S_{\mu\nu} - \frac{1}{2}G_{\mu\nu}S^\lambda_\lambda\right)\delta(\eta) + \text{non}-\text{singular terms}\;. \end{aligned} \tag{30}$$

The only non-zero contribution to the δ-function singularities on both sides of Eq. (30) arises for $(\mu\nu) = (\eta\eta)$. In order to avoid coordinate singularity on the horizon it is more convenient to consider the mixed component version of the latter:

$$R^\eta_\eta = 8\pi \left(S^\eta_\eta - \frac{1}{2} S^\lambda_\lambda \right) \delta(\eta) + \text{non} - \text{singular terms} \; . \tag{31}$$

Evaluating the l.h.s. of (31) through the formula:

$$R^r_r = -\frac{1}{2} \frac{1}{r^{D-2}} \partial_r \left(r^{D-2} \partial_r A \right) \tag{32}$$

valid for any spherically symmetric metric of the form (21) and recalling $r = r_0 + |\eta|$, we obtain the following matching condition for the coefficients in front of the δ-functions on both sides of (31) (analog of Israel junction conditions [8, 9]):

$$\partial_\eta \widetilde{A} \,\big|_{\eta \to +0} - \partial_\eta \widetilde{A} \,\big|_{\eta \to -0} = -\frac{16\pi\, \chi}{(2a_0)^{p/2-1}} \quad , \quad (\text{recall } D = p + 2) \; . \tag{33}$$

Eq. (33) yields a relation between the parameters of the spherically symmetric outer regions of "vacuum" solution (21) of Einstein Eqs. (25) and the dynamical tension of the *LL-brane* sitting at the (outer) horizon.

As an explicit example let us take (21) to be the standard $D = 4$ Reissner-Nordström metric, i.e., $A(r) = 1 - \frac{2m}{r} + \frac{e^2}{r^2}$. Then Eq. (33) yields the following relation between the Reissner-Nordström parameters and the dynamical *LL*-brane tension:

$$4\pi\chi\, r_0^2 + r_0 - m = 0 \quad , \quad \text{where} \;\; r_0 = m + \sqrt{m^2 - e^2} \; . \tag{34}$$

Eq. (34) indicates that the dynamical brane tension must be *negative*. Eq. (34) reduces to a cubic equation for the Reissner-Nordström mass m as function of $|\chi|$:

$$\left(16\pi\, |\chi|\, m - 1\right) \left(m^2 - e^2\right) + 16\pi^2 \chi^2 e^4 = 0 \; . \tag{35}$$

In the special case of Schwarzschild wormhole ($e^2 = 0$) the Schwazrschild mass becomes:

$$m = \frac{1}{16\pi\, |\chi|} \quad . \tag{36}$$

Let us observe that for large values of the *LL-brane* tension $|\chi|$, m is very small. In particular, $m \ll M_{Pl}$ for $|\chi| > M_{Pl}^3$ (M_{Pl} being the Planck mass). On the other hand, for small values of the *LL-brane* tension $|\chi|$ Eq. (34) implies that the Reissner-Nordström geometry of the wormhole must be near extremal ($m^2 \simeq e^2$).

5 Conclusions

In the present note we have first stressed two fundamentally important properties of *LL-brane* dynamics: (a) the *LL-brane* tension appears as a non-trivial additional dynamical degree of freedom instead of being given as an *ad hoc* constant; (b) *LL-branes* of codimension one automatically locate themselves on horizons of space-times with black hole type geometries.

Next, we have proposed self-consistent spherically symmetric solutions to the Einstein-Maxwell system which are traversable wormholes of Misner-Wheeler type and whose gravitational source are *LL-branes* located at their throats. Specifically we have considered Reissner-Nordström wormhole which is built by sewing together two outer regions of Reissner-Nordström space-time along the external horizon automatically occupied by the *LL-brane*, whose surface stress-energy tensor (derived from a consistent world-volume action principle) implements the pertinent Israel junction conditions. Thus, at the throat of the

Reissner-Nordström wormhole sits a neutral *LL-brane* with negative tension, the lines of force of the electric field go through the throat of the wormhole from one universe to the other, without any real electrical source anywhere, however in one universe there is the appearance of a positive charge, while in the other there is the appearance of a negative charge.

Finally let us note that according to Eq. (34) (in particular Eq. (36)) wormholes built from *LL-branes* with very high negative tension have a small mass.

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